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Surface plasmon polaritons in a topological insulator embedded in an optical cavity

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In recent years, topological insulators (TIs) have been theoretically predicted and experimentally observed.1–12 TIs experience a nontrivial rotation due to the jump in a quantized angle parameter because the condition of momentum conservation is not satisfied in the photon absorption process. Therefore, to detect this rotation, surface plasmons in a TI thin film embedded in an optical cavity. It is found that the frequencies of SPP modes are within the terahertz (THz) bandwidth and can be tuned effectively by adjusting the surface electron density and/or the optical cavity length. Since the surface electron density can be well controlled by the gate-voltage applied perpendicular to the TI surface, our theoretical results indicate that gated TI thin films may have potential applications in the electrically tunable THz plasmonic devices. © 2014 AIP Publishing LLC. [http://dx.doi.org/10.1063/1.4869228]

In recent years, topological insulators (TIs) have been theoretically predicted and experimentally observed.1–12 TIs with a single Dirac cone such as Bi2Te3 and Bi2Se3 are characterized by gapped insulating bulk states and gapless conducting surface states.8,9 The surface states in TIs are protected by the time-reversal (TR) symmetry and thus exhibit unique physical properties, i.e., they are topologically robust against TR invariant perturbations and their spin orientations are strictly determined by their momentum directions due to the spin-momentum locking.13,14 These exotic properties make TIs have potential applications in quantum computing and spintronic devices.

Recently, collective excitation modes associated with surface plasmons in the TI system have been theoretically investigated.15–18 Their dispersion relations exhibit the same wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wavevector dependency as those in graphene due to identical density wave in the TI system couple to the spin-density wave.

When the momentum conservation is achieved, the surface plasmons in the TI system can couple to the photons in the radiation field, giving rise to new collective modes called surface plasmon-polaritons (SPPs). Studying the SPPs can help to better understand the physical mechanisms behind their potential applications in practical devices. Further, the advantage of SPPs in comparison with surface plasmons for practical device applications is that their damping is less than plasmon damping. Based on these considerations, in this letter, we intend to investigate the SPPs in TI thin films fabricated from Bi2Se3. Our aim is to understand the tuning mechanism of SPPs in TI thin films and to explore their potential applications.

Here, we consider a TI thin film of Bi2Se3 embedded in an optical cavity, as shown in Fig. 1. If the film is thin enough (~1–10 nm), the surface states on the top and down surfaces can hybridize to open a gap at the Dirac point, and this hybridization gap has been observed by photoemission on thin films of Bi2Te3.22 The effective surface-state Hamiltonian of a TI thin film at the center of Brillouin zone can be written as

\[
H = \begin{pmatrix}
\Delta & \hbar v_F(k_x - ik_y) \\
\hbar v_F(k_x + ik_y) & -\Delta
\end{pmatrix},
\]

where \(v_F\) is the Fermi velocity and \(\Delta\) is the hybridization-induced surface gap. The energy spectrum and wavefunction for an electron in the conduction band (\(\lambda = 1\)) or a hole in the valence band (\(\lambda = -1\)) are given by \(E_{\lambda}(k) = \lambda \sqrt{\gamma^2 k^2 + D^2}\) and \(\Psi_{\lambda}(r) = |\lambda, k| e^{i\phi_B}, \phi_B = \frac{\theta_k}{2} - \frac{\pi \lambda}{2} k_x + \frac{\pi \lambda}{2} k_y\). Here, \(\gamma = \hbar v_F\) is the band parameter, \(r = (x, y)\) is the in-plane coordinates, \(k = (k_x, k_y)\) is the in-plane wavevector for a carrier and \(k = \sqrt{k_x^2 + k_y^2}\), \(\theta_k = x_k + (\lambda + 1)\pi / 2\) with tan \(x_k = \gamma k / \Delta\), and \(\phi_k\) is the angle between \(k\)
and the x-direction and tan $\phi_x = (k_x/k_l)$. In this work, we will consider a n-type TI thin film with nonzero chemical potential $\mu$. When $\mu$ lies inside the bulk energy-gap, a Dirac-type two-dimensional electron gas (2DEG) with nonzero density $N$ can be formed in the surface of TI thin film. The collective oscillation of such 2DEG gives rise to the 2D surface plasmon in the TI thin film.

In the direction perpendicular to the TI thin film, the optical cavity (see Fig. 1) serves as a resonator for light and due to the presence of such a cavity, the photon energies for quantized cavity modes can be solved from the Maxwell’s equation and are given by:

$$\hbar \omega_0 = \frac{hc}{\sqrt{\epsilon}} \sqrt{p^2 + \left(\frac{m\pi^2}{L^2}\right)^2}. \quad (2)$$

Here, $\omega_0$ is the photon frequency, $p = |p|$ with $p$ being the in-plane wavevector of a photon, $m = 1, 2, 3, \ldots$ is the quantum number of cavity modes, $L$ is the optical cavity length, $c$ is the light speed in vacuum, and $\epsilon$ is the dielectric constant of medium where light travels. Using the equality of the energy of photon to the volume energy of the EM filed in the cavity and the relationship between the transverse ($F_0$) and longitudinal ($F_z$) components of the electric field $[F_0/F_z = pL/(m\pi)]$, we can obtain the intensity of quantized EM field as $F_0^2 = 8\hbar \omega_0 p^2 L^2/(\epsilon S L (2p^2 L^2 + m^2 \pi^2))$ with $S$ being the sectional area of the cavity parallel to the surface of TI thin film. By changing the optical cavity length one can achieve the condition of resonant interaction between the 2D photon with the momentum $h\mathbf{p}$ and the 2D plasmon with the momentum $h\mathbf{q}$, namely, $p = q$.

With the TI surface states and the cavity photon modes obtained, we can calculate the SPP modes induced by the plasmon-photon coupling in the considered system.

Applying the electron wavefunctions to the electron-electron (e-e) interaction Hamiltonian induced by the Coulomb potential, the space Fourier transform of the matrix element for bare e-e interaction can be written as

$$V_{ee}(k, q) = V_q |F(k, q)|^2, \quad (3)$$

where $V_q = 2\pi \epsilon^2/(\epsilon q)$. $|F(k, q)|^2 = \frac{1 + \cos k_x q \cos k_y + \sin k_x q \sin k_y B_{kq}}{2}$, $B_{kq} = \frac{(k + q \cos \phi)/|k + q|}$, and $\phi$ is the angle between $k$ and $q$.

The EM field in the plane of TI thin film is completely described by the vector potential $A = (A_x, A_y) |q|^2$, where $\phi$ is the angle between $A$ and the x direction and $A = F_0/\omega_0$. The electron-photon (e-p) interaction Hamiltonian is obtained by the substitution $k \rightarrow k + e\mathbf{A}/\hbar$, which leads to

$$H_{ep} = \left(\frac{\gamma eF_0}{\hbar \omega_0}\right) \left[\begin{array}{cc} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{array}\right] q. \quad (4)$$

The space Fourier transform of the matrix element for bare e-p interaction can be written as

$$V_{ep}(k, q, \Omega) = SD_0(\omega_0, \Omega)|U_{ep}(k, q)|^2, \quad (5)$$

where $\Omega$ is the excitation frequency, $D_0(\omega_0, \Omega) = 2\hbar \omega_0/[(\hbar \Omega)^2 - (\hbar \omega_0)^2]$ is the bare photon propagator, $|U_{ep}(k, q)|^2 = |\langle \hat{\mathbf{j}}, k | H_{ep} | \hat{\mathbf{j}}, k + q \rangle|^2 = W_{ep}^2 (G(k, q))^2$ is the squared matrix element for the e-p interaction, $W_{ep} = \gamma eF_0/\hbar \omega_0$ is the e-p coupling coefficient, $|G(k, q)|^2 = (1 - \cos k_x q \cos k_y \pm \sin k_x q \sin k_y B_{kq})/2$ with signs + and − representing the electron interaction with TM-polarized (A $\perp$ q) and TE-polarized (A $\parallel$ q) EM waves, and the angular factor $C_{kq} = |k \cos 2(\phi - \phi) + q \cos (\phi - \phi)|/(|k + q|)^2$.

In Ref. 25, the effective e-e interaction in the presence of electron-phonon interaction was derived in the random phase approximation (RPA) or bubble-diagram approximation. Since photons and phonons have similarities, i.e., both of them satisfy the Bose statistics, in this work we employ the same approach to derive the effective e-e interaction in the presence of electron-phonon coupling, from which the dynamical dielectric function in the RPA can be written as

$$\varepsilon(q, \Omega) = 1 - g_s \sum_k [V_{ee}(k, q) + V_{ep}(k, q, \Omega)] \Pi(k, q, \Omega). \quad (6)$$

Here, $g_s = 2$ accounts for the spin degeneracy and $\Pi(k, q, \Omega)$ is the density-density correlation function or polarization function which is central quantity needed to describe the dielectric properties of an electronic system and is given by

$$\Pi(k, q, \Omega) = \frac{f[E_s(k)] - f[E_s(k + q)]}{\hbar \Omega + E_s(k) - E_s(k + q)} \quad (7)$$

with $f(E)$ being the Fermi-Dirac function. Due to the in-plane symmetry, we will see that after performing the integral over $k$, the final result of $\varepsilon(q, \Omega)$ is independent of the angle $\phi$. 

FIG. 1. TI thin film of Bi2Se3 grown on the substrate Al2O3 embedded in an optical cavity. Here, the characters “SP” and “SPP” represent the surface plasmons and surface plasmon polaritons propagating along the surface of TI thin film.
This means that the dynamical dielectric properties of an 2D electronic system with the in-plane symmetry are regardless of the polarization direction of the EM field. The SPP modes are determined by \( \epsilon(q, \Omega) = 0 \).

Now, we present the numerical results and some discussions on the SPPs in the n-doped TI thin film embedded in the optical cavity. In the present work, we only consider the SPPs that originate from the interaction of surface plasmons with TM-polarized EM waves. Similar results can be found for the case of TE-polarized ones. The main parameters used in the calculation are taken from Ref. 20, which are, respectively, the Fermi velocity \( v_F = 6 \times 10^7 \) cm/s, and the dielectric constant \( \epsilon = 5.5 \). The hybridization gap is taken as \( \Delta = 0 \) meV, which means the gapless surface states in the TI thin film. Other important parameters, including the surface electron density \( N \) and the optical cavity length \( L_o \), are set to be adjustable to study their influences on the SPP modes. The bulk energy-gap of Bi\(_2\)Se\(_3\) is about 0.3 eV, as confirmed by the photoemission experiment. 8 This means that \( N \) can reach up to about \( 3.6 \times 10^{13} \) cm\(^{-2}\). Therefore, the surface electron density can be tuned in a wide range, i.e., by the gate-voltage control of the chemical potential. 26,27

The bulk energy-gap of Bi\(_2\)Se\(_3\) is about 0.3 eV, as confirmed by the photoemission experiment. 8 This means that \( N \) can reach up to about \( 3.6 \times 10^{13} \) cm\(^{-2}\). Therefore, the surface electron density can be tuned in a wide range, i.e., by the gate-voltage control of the chemical potential. 26,27 Furthermore, to achieve the efficient coupling between the surface plasmons and cavity photons, we only consider the lowest cavity mode, i.e., the quantum number \( n = 1 \).

In Figs. 2(a) and 2(b), we show the dispersion relations for decoupled modes (surface plasmon and photon modes) and coupled modes (SPP modes) at fixed temperature, optical cavity length and surface electron density as indicated. One can see that the plasmon-photon coupling causes two Rabi splittings (anticrossing behaviors) between the upper and lower branches of SPP modes. The Rabi splitting reflects the resonant interaction between the surface plasmons in the TI thin film and the photons in the optical cavity. It is clear that the resonant frequencies around two Rabi points are within the THz bandwidth. This implies that the THz EM waves can propagate along the surface of a TI thin film.

To examine how the surface electron density and the optical cavity length affect the SPP modes, we calculate the \( N \)- and \( L \)-dependencies of SPP modes and show the corresponding results in Figs. 3(a) and 3(b). From the figure, we can observe that there are different numbers of the Rabi splitting for different \( N \) or \( L \). For instance, there are no, one, and two Rabi splittings for small, moderate, and larger \( N \), respectively. Also, the frequencies of SPP modes change significantly with varying \( N \) or \( L \). These results indicate that the single- or dual-mode propagation of THz EM waves along the TI surface can be easily realized and the frequencies of these modes can be effectively tuned by adjusting the surface electron density and/or the optical cavity length.

Finally, we discuss some interesting features of SPPs in TI thin films, which are not present in other ordinary 2D materials. Since the surface states in the TI are protected by the TR symmetry and thus topologically robust against the TR invariant perturbations, including the temperature variation, the SPP modes in the system are also topologically robust against these perturbations when they propagate along the TI surface. Therefore, the energy loss during the SPP propagation would be reduced. Furthermore, such coupled collective modes may preserve the coherence of TI surface states up to room temperature. This implies that TI thin films may find potential applications in room temperature plasmonic devices, which would be a great advantage of TIs over other materials.

Before closing this paper, we should mention a recent theoretical study on the spin-charge separation of plasmonic excitations in a thin slab of TIs. 28 In that work, the authors argued that to explain the recent experimental findings 20 it is necessary to include the 2DEG formed in the depletion layer due to the band bending at the two surfaces. Taking such an effect into account, the number and behavior of surface plasmon and thus SPP mode in a TI thin film would be changed. However, as pointed out in Ref. 28, due to the closeness of the depletion layer to the surface, the newly emerging modes are closely pinned to the particle-hole continuum and...
probably not observable. Therefore, in the present work, we have neglected the effect of the depletion layer on the SPPs in the TI thin film and only focused on the interactions between the Dirac-type surface plasmons and photons.

In summary, we have theoretically investigated the SPPs induced by the plasmon-photon coupling in a TI thin film embedded in an optical cavity. We have found that the frequencies of SPP modes are within the THz bandwidth and can be tuned effectively by adjusting the gate-controlled surface electron density and/or the optical cavity length. Our theoretical results are interesting for both the fundamental physics and potential applications of TI-based plasmonics.

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24Here, we directly transfer the result of bare phonon propagator in the presence of electron-phonon coupling [see, for example, R. Jalabert and S. Das Sarma, Phys. Rev. B 40, 9723 (1989)] to the case of a photon gas confined in an optical cavity. This is because (1) the photon modes are quantized in the optical cavity, similar to the phonon modes in the lattice (the quantization of lattice vibration) and (2) both the photons and phonons are bosons satisfying the Bose statistics.