Fast-electron optical spectrum of a two-dimensional electron gas in the presence of the Rashba effect☆

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Abstract

We examine how the Rashba spin-orbit interaction (SOI) affects the fast-electron optical spectrum of a two-dimensional electron gas (2DEG). It is found that for a spin-split 2DEG, the spectrum of optical absorption is mainly induced by plasmon excitation via inter-SO electronic transition. From the width and position of the spectrum, the Rashba spin-splitting can be identified optically and, therefore, important spintronic properties can be measured through optical experiments.

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1. Introduction

At present, most of the published work on spintronic systems realized from narrow-gap semiconductor quantum well structures, such as InGaAs/InAlAs-based two-dimensional electron gases (2DEGs), has been focused on electronic and transport properties of the devices. This is mainly owing to potential applications in future quantum computation and communication. One of the most popularly used experimental techniques to identify the Rashba spin-splitting in an InGaAs-based 2DEG is magnetotransport measurements carried out under the condition where the Shubnikov-de Hass (SdH) oscillations are observable. From the periodicity of the SdH oscillations, the electron density in different spin branches can be obtained and, from this result, the Rashba parameter can be determined [1]. However, from a technical point of view, there are several drawbacks in using the magnetotransport experiments to study the spintronic properties. Firstly, relatively high magnetic fields $B$ are required to be able to observe the SdH oscillations, especially for high-density samples. Secondly, other spin effects occurred at $B\neq 0$ (e.g., Zeeman splitting) may mix with the Rashba effect. Thirdly, Ohmic contacts have to be made on the sample to conduct the transport measurements. Hence, in order to carry out sample characterization more accurately and easily, it is more favorable to use optical experiments for the measurement of the spintronic properties. In this paper, we examine how spin-orbit interaction (SOI) can affect the optical and optoelectronic properties of a spintronic system. Our prime motivation of this study is to see whether the spectrum of optical absorption induced by elementary electronic excitation can be used to determine the spintronic properties of a 2DEG in the presence of the Rashba SOI.

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2. Outline of theoretical approaches

For a 2DEG formed in the xy-plane, the Schrödinger equation including the Rashba SOI can be solved analytically. Applying the electron wavefunctions to the electron–electron (e–e) interaction induced by the Coulomb potential, we can calculate the electrostatic energy caused by bare e–e interaction. Using the energy spectrum of a spin-split 2DEG, the electron density–density (d–d) correlation function \( P_{\sigma\sigma}(\Omega, q) \) can be obtained [2] in the absence of e–e screening, where \( \sigma = \pm 1 \) refers to different spin branches, \( \Omega \) is the excitation frequency, and \( q \) is the change of electron wavevector during an e–e scattering event. Thus, under the random-phase approximation (RPA), the dynamical dielectric function matrix \( \epsilon'_\sigma(\Omega, q) \) can be obtained, where \( \nu = (\sigma', \sigma) \). It should be noted that for a spin-split 2DEG, there are four channels for electronic transition [\( i.e., \nu = (\sigma', \sigma) = (\pm, \pm), (\pm, \mp), (\mp, \pm), (\mp, \mp) \)] and \( (\pm, -) \). Hence, \( \epsilon'_\sigma(\Omega, q) \) is a \( 4 \times 4 \) matrix [3]. From the real part of the determinant of \( \epsilon'_\sigma(\Omega, q) \), the plasmon modes can be determined by taking \( \text{Re}\{\epsilon'_\sigma(\Omega, q)\} \rightarrow 0 \). At the low-temperature (\( i.e., T \rightarrow 0 \)) and long-wavelength (\( i.e., q \rightarrow 0 \)) limit, the plasmon frequencies induced by intra- and inter-SO electronic transition are given, respectively, by

\[
\Omega_0 = \omega_p [1 - 2(\omega_- - \omega_+)]/\omega_0^{1/2},
\]

and solving

\[
\ln \frac{\Omega + \omega_-}{\Omega - \omega_-} = \frac{\omega_0 \Omega}{\omega_p^2} = \frac{\omega_0 \Omega}{\omega_0 \Omega}.
\]

Here, \( \omega_p = (2\pi e^2 n_g q/(\kappa m^*)^{1/2} \) is the plasmon frequency of a spin-degenerate 2DEG, \( \kappa \) is the dielectric constant of the material, \( m^* \) is the electron effective-mass, \( n_g = n_+ + n_- \) is the total electron density of the 2DEG with \( n_\pm \) being the electron density in the \( \pm \) spin branches, \( \omega_\pm = 4\pi x\sqrt{n_\pm}/h \) with \( x \) being the Rashba parameter which measures the strength of the SOI, and \( \omega_0 = 16\pi n_e h/m^* \).

Using the Kubo formula in which the current–current correlation is induced by electron interactions with the radiation field, the optical conductivity for a spin-split 2DEG can be calculated through \[ \sigma(\Omega) = -\lim_{q \to 0} \frac{e^2}{q^2} \sum_{\nu} \text{Im} \Pi_{\nu}(\Omega, q), \]

where \( \Pi_{\sigma\sigma}(\Omega, q) \) is the electron d–d correlation function in the presence of e–e screening, which reads

\[
\Pi_{\nu}(\Omega, q) = \sum_{\sigma'} \epsilon_{\nu\sigma'}^{-1}(\Omega, q) P_{\sigma\sigma}^0(\Omega, q),
\]

where \( \epsilon_{\nu\sigma'}^{-1}(\Omega, q) \) is the inverse dielectric function matrix. It should be noted that because the electron–photon interaction normally does not change the wavevector of an electron, \( q \) has to be small in Eq. (3). We find \( \lim_{q \to 0} q^{-2} \text{Im} \Pi_{\nu++}(\Omega, q) = \lim_{q \to 0} q^{-2} \text{Im} \Pi_{\nu--}(\Omega, q) = 0 \), which implies that the intra-SO transition does not contribute to optical spectrum. In contrast, a strong optical absorption can occur via inter-SO transition, especially for a transition channel from a lower ‘–’ spin branch to a higher ‘+’ branch. When \( q \to 0 \) and \( T \to 0 \), the optical conductivity induced by inter-SO transition is obtained as

\[
\sigma(\Omega) = (e^2/\hbar) \Theta(\omega_- - \Omega) \Theta(\Omega - \omega_+) \times \left[ \frac{2(1 - a_0^2 + 2\alpha_0^2)}{(1 - a_0 - a_1)^\nu + a_2^2} - 1 \right],
\]

where \( a_0 = \gamma \ln[(\omega_+/\omega_-)/(\Omega + \omega_-)], \quad a_1 = \gamma \ln[(\omega_-/\omega_+)/(\Omega - \omega_-)], \quad a_2 = \gamma \Theta(\omega_- - \Omega) \Theta(\Omega - \omega_+), \) and \( \gamma = \omega_0^2/\omega_0 \Omega \). For \( T \to 0 \) the electron density in the \( \pm \) spin channel is [3]: \( n_\pm = (n_\pm/2) \mp (k_z/2\pi) \sqrt{2\pi n_e - k_z^2} \) with \( k_z = m^* x^2/h^2 \), which results in \( \omega_- - \omega_+ = 4m^* x^2/h^3 \).

From Eq. (5), we see immediately that the edges of the spectrum are, respectively, at \( \Omega = \omega_\pm = 4\pi x\sqrt{n_\pm}/h \) and the width of the spectrum is \( \omega_- - \omega_+ = 4m^* x^2/h^3 \).

3. Results and discussions

In this paper, we consider an InGaAs/InAlAs-based 2DEG in which a strong Rashba spin-splitting has been observed experimentally [1,4]. It has been found experimentally that when total electron density \( n_g \sim 10^{11} \) cm\(^{-2} \), the Rashba parameter \( x \) can reach up to \( 3 \sim 4 \times 10^{-11} \) eVm [1,4]. These typical sample parameters are taken within the numerical calculations. We find that in a spin-split 2DEG, there are three plasmon branches induced by intra- and inter-SO electronic transition. The dependence of plasmon frequency due to intra- and inter-SO excitation on \( \omega_p \sim q^{1/2} \) is shown in Fig. 1 for fixed Rashba parameter \( x \) and

![Fig. 1. Plasmon frequency as a function of \( \omega_p = (2\pi e^2 n_g q/(\kappa m^*))^{1/2} \) for fixed Rashba parameter \( x \) and electron density \( n_\pm \). Here, \( \Omega_0 \) and \( \Omega_\pm \) are induced, respectively, by intra- and inter-SO excitation, and \( \omega_\pm = 4\pi x\sqrt{n_\pm}/h \).](image-url)
electron density \( n_e \). We see that over a wide regime of \( \omega_p \) or \( q \), the plasmon frequency induced by inter-SO transition \( \Omega_\pm \rightarrow \omega_\pm \approx 4x\sqrt{nn_\pm}/\hbar \) depends very little on \( q \). Hence, when \( q \rightarrow 0 \) inter-SO plasmons are optic-like, in contrast to intra-SO plasmons for which \( \omega_\pm \sim q^{1/2} \) (also see Eq. (1)).

The results shown in Fig. 1 and in Eq. (5) indicate that the shape and profile of the optical spectrum for a spin-split 2DEG is determined mainly by inter-SO plasmon excitations which are optic-like and with frequencies \( \omega_{\pm} \). The dependence of optical conductivity on total electron density \( n_e \) and the Rashba parameter \( \alpha \) is shown, respectively, in Figs. 2 and 3. It is known that the optical absorption coefficient is proportional to \( \sigma(\Omega) \). \( \sigma(\Omega) \) can therefore represent basic features of the optical absorption spectrum. We see that for a spin-split 2DEG, the optical spectrum induced by e–e interaction via plasmon excitation takes roughly a rectangular shape. Because optical absorption is achieved mainly via inter-SO electronic transition, the position and the width of the spectrum are determined by the frequencies of two optic-like plasmon modes \( \omega_{\pm} \). When \( q \rightarrow 0 \), \( \omega_{\pm} \sim q \ll \Omega_0 \) so the influence of the e–e screening on the shape of the spectrum is rather weak. As a result, the amplitude of optical conductivity is roughly a universal value \( \sigma(\Omega) \approx c^2/16\hbar \) (see Eq. (5)).

From Fig. 2, we note that at a fixed Rashba parameter \( \alpha \), the blue-shift of the spectrum can be achieved via increasing the total electron density \( n_e \), because \( \omega_{\pm} \) increases with \( n_e \). However, varying the electron density of a sample does not change the width of the spectrum because \( \omega_\pm \sim \omega_\pm \) does not depend on \( n_e \) (see Eq. (6)). From Fig. 3, we find that the strength of SOI affects strongly both the position and the width of \( \sigma(\Omega) \). At a fixed \( n_e \), the increase in \( \alpha \) leads to a blue-shift and to a broadening of the optical spectrum, because \( \omega_{\pm} \) increases with \( \alpha \) and their difference depends on \( \alpha^2 \) (see Eq. (6)). Our numerical results indicate that when \( \omega_\parallel < 1 \text{THz} \), \( \sigma(\Omega) \) depends very little on the value of \( \omega_p \), similar to the dispersion relation of the inter-SO plasmon modes \( \Omega_\pm \) shown in Fig. 1.

An important conclusion we can draw from these theoretical results is that if we can measure optical spectrum of a spin-split 2DEG, important spintronic coefficients can be obtained optically. From the width of the spectrum, we can determine the value of the Rashba parameter using Eq. (6). From the position or edges of the spectrum, we can obtain two plasmon frequencies \( \omega_{\pm} \approx 4x\sqrt{nn_\pm}/\hbar \) and then the electron densities \( n_\pm \) in the “±” spin orbits. Thus, the total electron density \( n_e = n_\pm + n_- \) and the spin polarization \( P = (n_+ - n_-)/n_e \) can be determined straight away. Consequently, the technical drawbacks of using magnetotransport experiments for the determination of the spintronic properties can be overcome by using optical experiments.

Recent photoconduction measurements on spin-split 2DEGs have shown that spin-induced optical absorption does occur in InGaAs-based quantum well systems [5]. In these experiments, the response of spin-split electrons to the radiation field is detected by transport measurements (i.e., the Ohmic contacts are still required) via cyclotron resonances (i.e., the magnetic fields are present). Optical spectra induced plasmon excitation in spin-degenerate 2DEG systems have been observed using techniques such as optical absorption or transmission [6], Raman spectrum [7], ultrafast pump-and-probe experiments [8], etc. On the basis that the optical spectrum of a spin-split 2DEG is mainly induced by optic-like plasmons via inter-SO transition, the optical spectrum of a spin-split 2DEG can also be measured using these state-of-the-art experimental techniques. In particular, it is known that the plasmon frequency of a spin-degenerate 2DEG depends on changes of electron wavevector via \( \omega_\parallel \sim q^{1/2} \). To measure the optical

![Fig. 2](image1.png) Optical conductivity \( \sigma(\Omega) \) as a function of excitation frequency \( \Omega \) at a fixed Rashba parameter \( \alpha \) for different electron densities \( n_e \) as indicated.

![Fig. 3](image2.png) \( \sigma(\Omega) \) vs. \( \Omega \) at a fixed \( n_e \) for different values of \( \alpha \) as indicated. \( \omega_\parallel = 4x\sqrt{nn_\parallel}/\hbar \).
spectrum of such a system, the techniques such as grating couplers are needed [8]. However, the plasmon frequencies \( \omega_{pl} \) induced by inter-SO transition in a 2DEG with the Rashba SOI are optic-like and well defined. Hence, optical spectrum of a spin-split 2DEG can be more easily measured optically.

In this work, we have found that the optical spectrum induced by e–e interaction in a spin-split 2DEG relates directly to important spintronic coefficients. Through examining the position and the width of the fast-electron optical spectrum, these spintronic coefficients can be determined easily and accurately. We have proposed a way to measure optically the spintronic coefficients such as the Rashba parameter, electron density in different spin orbits, spin polarization, etc. As a conclusion, we believe that the Rashba spin-splitting in a narrow-gap 2DEG can be identified optically and the drawbacks of the sample characterization using magnetotransport experiments can be overcome. We hope that the important and interesting theoretical predictions in this paper merit attempts at experimental verification.

References