Nonadiabatic quantum spin pump: Interplay between spatial interference and photon-assisted tunneling in two-dimensional Rashba systems

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(Received 14 August 2006; revised manuscript received 10 October 2006; published 9 January 2007)

We investigate the nonadiabatic transport property of parametric quantum spin pumps in the presence of Rashba spin-orbit (SO) interaction. It is well known that in the adiabatic regime, the pumped charge and spin currents are zero when all the pumping parameters are in phase. But the pumped charge and spin currents can be nonzero with nonadiabatic contribution and spatial interference. In such a case, we have analytically proved the following: (i) If the potential including the pumping potential has the symmetry \( V(x,y,t) = V(-x,y,t) \) or \( V(x,y,t) = V(-x,-y,t) \), where \( x \) is along the direction of pumped current, then the pumped charge current is zero but the pumped spin current may not be zero; (ii) if the potential including the pumping potential has the symmetry \( V(x,y,t) = V(x,-y,t) \), then the pumped spin currents with spin along the \( x \) and \( z \) directions are zero. Since the phases of pumping potential are very easy to control, this provides a robust way of generating pure spin current. Numerically, we have verified the analytic results and calculated the pumped charge and spin current in the nonadiabatic regime at finite pumping frequency and finite pumping amplitude as well as external bias. In general, both charge current and spin current are pumped out of the system in the presence of SO interaction. Since both pumped charge current and spin current depend sensitively on the system parameters, the pure spin current without accompanying charge current can be produced by controlling system parameters such as external bias, phase difference between two pumping amplitudes, and the pumping frequency. An interesting interplay between spatial interference and photon-assisted tunneling processes is observed.

DOI: 10.1103/PhysRevB.75.035312 PACS number(s): 73.63.--b, 72.10.Bg, 72.25.--b, 85.75.--d

I. INTRODUCTION

Spin current is of both fundamental interest and practical importance in the emerging field of spintronics. One of the key issues in this field is how to generate spin current in spintronic devices. At present, many methods have been proposed. For instance, the recently predicted spin-Hall effect in spin-orbit (SO) coupled systems has become an effective way of generating dissipationless spin current. The approaches using magnetic components for producing spin current include utilizing static magnetic field, ferromagnetic material, or ac magnetic field. Generating spin current by optical means is suggested to use coherently controlled optical excitations between the valence and the conduction band and is observed experimentally in bulk crystals and quantum wells. Most methods currently under investigation require strong magnetic field and interfaces between semiconductors and ferromagnets, which is very inefficient in practice. A parametric spin pump in SO systems is an alternative to avoid this. It is considered a promising approach and has attracted considerable interests.

A parametric quantum pump is a device that generates the flow of an electron by cyclic variations of system parameters. In the presence of an external magnetic field or SO interaction, the pumped current is spin-polarized, giving rise to a pumped spin current. Many types of parametric spin pumps have been proposed, and an observation of the pumped spin current has been reported using magnetic means. However, pumping spin current using solely an electrical field is more desirable. In the adiabatic regime, a few spin pumps have been proposed to utilize the time-dependent gate to pump spin current in SO coupled systems, either by modulating the shape of a quantum dot or by modulating the strength of the SO coupling constant. Less attention has been given to the nonadiabatic regime where the pumping frequency is finite and the system is far away from equilibrium. It is the purpose of this paper to fill this gap. In this paper, we present a theory for a nonadiabatic spin pump in the presence of Rashba SO interaction, assuming that the full potential of the system consists of spin independent potential \( V(x,y,t) \) and SO interaction \( V_{SO} \). When all the pumping parameters are in phase, we have proven the following: (i) If the potential including the pumping potential has the symmetry \( V(x,y,t) = V(-x,y,t) \), where \( x \) is along the direction of pumped current, the pumped charge current is zero. The spin currents with spin along the \( y \) and \( z \) directions are conserved. The spin currents from both leads with spin along the \( x \) direction are either flowing into or out of the quantum pump. (ii) If the potential including the pumping potential has the symmetry \( V(x,y,t) = V(x,-y,t) \), the pumped spin currents with spin along the \( x \) and \( z \) directions are zero. (iii) If the potential including the pumping potential has the symmetry \( V(x,y,t) = V(-x,-y,t) \), the pumped charge current is zero. The spin currents with spin along the \( x \) and \( y \) directions are conserved. The spin currents from both leads with spin along \( z \)-direction are either flowing into or out of the quantum pump. Physically, the reason that pure spin current...
can be generated in the presence of SO interaction is the following. For a system with external bias, the direction of current is fixed. For the parametric pumping, the direction of pumped current is not known in advance since there does not exist a preferred direction such as the electric field (due to the external bias). It is a well-known fact that the direction as well as the magnitude of the pumped current are very sensitive to various parameters of the system, such as potential landscape of the pump,\textsuperscript{30,31} pumping frequency,\textsuperscript{32} and Fermi energy of the leads.\textsuperscript{28} Because the electron with different spins may experience different system parameters in the presence of SO interaction, it is possible that spin-up electrons are pumped out of the system in the presence of external bias in the leads. Since there is no spin-orbit interaction in the lead, the spin index is a good quantum number. Starting from the standard expression of time-dependent current in terms of Keldysh nonequilibrium Green’s functions,\textsuperscript{33} the particle current with spin $\sigma$ from the left lead can be written as

$$J_{L\sigma}(t) = -\frac{1}{\tau} \int_{0}^{\tau} dt \left[ \text{Tr}[G(t,t_1)\Sigma_{L}^{-}(t_1,t)] + G^{-}(t_1,t)\Sigma_{L}^{\sigma}(t_1,t) + \text{c.c.} \right],$$

(4)

where $\sigma = \uparrow$ or $\downarrow$ and the trace is over orbital degrees of freedom. Here $G^+$ and $G^-$ are the retarded Green function and the lesser Green’s function, respectively. $\Sigma_{L}^{\sigma} = i\Gamma_{L} f_{L}$ is the lesser self-energy determined by the linewidth function $\Gamma_{L}$ and Fermi distribution function $f_{L}$ of the left lead. The average particle current $J_{L\sigma}$ from the left lead during a pumping cycle can be written as

$$J_{L\sigma} = -\frac{1}{\tau} \int_{0}^{\tau} dt \left[ \text{Tr}[G(t,t_1)\Sigma_{L}^{-}(t_1,t)] + G^{-}(t_1,t)\Sigma_{L}^{\sigma}(t_1,t) + \text{c.c.} \right],$$

(5)

where $\tau = 2\pi/\omega$ is the period of pumping cycle. Carrying out a double-time Fourier transform

$$G^\gamma(t,t') = \frac{1}{2\pi} \int \frac{dE dE'}{2\pi} e^{iE t - iE' t'} G^\gamma(E,E'),$$

(6)

with $\gamma = r,a,\cdots,$ Eq. (5) becomes

$$J_{L\sigma} = -\frac{1}{(2\pi\tau)} \int \frac{dE dE'}{2\pi} \text{Tr}[\{G^\gamma(E,E') - G^\gamma(E,E')\} \times \Sigma_{L}^{\sigma(E,E')} + G^{-}(E,E') \times [\Sigma_{L}^{-}(E,E') - \Sigma_{L}^{\sigma(E,E')} ]\sigma,,$$

(7)

where we have extended the integration range for $dt$ in Eq. (5) to $[-N\tau,N\tau]$ with $N \to \infty$. We have also used the fact that the $\Sigma$ is a diagonal matrix in spin space because there is no SO interaction in the leads. Denoting $G^\gamma$ the retarded Green’s function with matrix element $G^E(E,E')$, we have $G^\gamma = -iG^* G\epsilon^\gamma$, where $\Gamma = \Gamma_{L} + \Gamma_{R}$. Since there is no time-dependent potential in the leads, we have $\Sigma_{L}^{\sigma(E,E')} = 2\pi \delta(E - E')\Sigma_{L}^{\sigma}(E)$ with $\alpha = L,R$. Using the Keldysh equation $G^\gamma = G^\Sigma G\epsilon^\gamma$, we obtain ($\hbar = 1$)

$$J_{L\sigma} = -\frac{1}{(2\pi\tau)^2} \int \frac{dE}{2\pi} \sum_{n=0}^{\infty} \text{Tr}[G^\gamma(E,E+n\omega) \times \Gamma_{L} G^\gamma(E+n\omega,E)\Gamma_{L} f_{L} - G^\gamma(E,E+n\omega) \times (\Gamma_{L} f_{L} + \Gamma_{R} f_{R}) G^\gamma(E+n\omega,E)\Gamma_{L} ]\sigma,$$

(8)

with $\Gamma_{R} = \Gamma(E+n\omega)$, $\Gamma_{L} = -2\text{Im} \{\Sigma_{L}^{\sigma}(E)\}$, $f_{\sigma}(E+n\omega - qV_{\sigma})$, and $V_{\sigma}$ is the external dc bias. In the absence of SO interaction and external bias voltage ($f_{\sigma} = f_{\Re}$), Eq. (8) is consistent with the general expression for charge pump.\textsuperscript{28,30} In order to get a clear physical insight, we rewrite Eq. (8) in the following form:
where $R_n(E) = \Gamma_n(E) G_n'(E) \Gamma_n^\dagger(E) G_n^\prime(E)$ is regarded as a reflecting term, \cite{36} $T_n(E)=\Gamma_n(E) G'_n(E) \Gamma_R(E) G_n^\prime(E)$ is regarded as a transmitting term, and $G_n^\prime(E)=G(E,E+\hbar \omega)$. In this expression, the summation over integer $n$ clearly indicates the contributions from the multiphoton process toward the pumped current. The charge current and spin current in the left lead are defined as

$$ I_L^q = q/(2J)_L, \quad I_L^s = \hbar/(2(J)_L - (J)_{-L}) \quad (10) $$

with spin matrices

$$ \sigma_z = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right), \quad \sigma_x = \hbar/(2) \left( \begin{array}{cc} 1 & 1 \\ -1 & -1 \end{array} \right). $$

Then the expression for charge (spin) current is

$$ I_L^q = \frac{1}{(2N)^2} \int \frac{dE}{2\pi} \sum_{n=-\infty}^{+\infty} \text{Tr}[\sigma_z R_n(E)(f_{Ln} - f_{L})] $$

$$ + \sigma_{e\pm} T_n(E)(f_{Rn} - f_{L}) \quad (12) $$

In order to obtain a closed-form solution for the pumped charge current and spin current, we need the expression of $G_n'(E)$, which has been derived in Ref. \cite{28}. Denoting $G^{0q}(E)$ the equilibrium Green’s function when the time-dependent pumping potential is set to zero, the multiphoton Green’s function satisfies the following equation:

$$ G_n'(E) = (G_{n+1}V_p^+ + G_{n-1}V_p)G^{0q}(E)/2. $$

Thus $G_n'(E)$ can be calculated iteratively from equilibrium Green’s function $G^{0q}(E)$, where

$$ G^{0q}(E) = \frac{1}{E - H_0 - \Sigma'} \quad (14) $$

with $\Sigma'$ the self-energy of the leads.

Now we use Eqs. (12) and (13) to calculate the pumped charge and spin currents for a 2D system with Rashba SO interaction. The tight-binding version of the Hamiltonian $H_0$ of the central scattering region in the absence of pumping potential is

$$ H_0 = \sum_{n\sigma} \epsilon_{n\sigma} c_{n\sigma}^\dagger c_{n\sigma} - \sum_{\langle nm \rangle \sigma} t_{\langle nm \rangle} c_{n\sigma}^\dagger c_{m\sigma} + t_{SO} \sum_n \left[ (c_{n\uparrow}^\dagger c_{n+\delta y \uparrow} - c_{n\downarrow}^\dagger c_{n+\delta y \downarrow}) - i(c_{n\uparrow}^\dagger c_{n+\delta y \downarrow} + c_{n\downarrow}^\dagger c_{n+\delta y \uparrow}) \right] + H.c.) \quad (15) $$

where $\epsilon_{n\sigma}$ is the on-site energy of an electron with spin $\sigma$ at site $n$; $n$ and $m$ are nearest neighbors; $t$ is the hopping coupling constant; $\delta_y$ and $\delta_x$ are unit vectors along the $x$ and $y$ directions; and $t_{SO}$ the SO coupling strength, satisfies $t_{SO} = \alpha t_0/2a_0$, with $a_0$ the lattice constant. For the discrete semi-infinite leads, using the transfer matrices of the leads, the self-energies can be obtained by calculating the surface Green’s function.\cite{37} The equilibrium Green’s function $G^{0q}$ can be calculated by matrix inversion. Then the multiphoton Green’s function $G_n'(E)$ can be calculated recursively from $G^{0q}$ by solving Eq. (13), and finally the pumped charge and spin current can be obtained from Eq. (12). When there is no external bias in the leads, the Fermi distribution function $f_{Ln}$ can be obtained from Eq. (9) can be rewritten as

$$ I_L^{q+} = \frac{1}{(2N)^2} \int \frac{dE}{2\pi} \sum_{n=-\infty}^{+\infty} A_n^{q+}(E)(f_{Ln} - f_{L}) \quad (16) $$

with $A_n^{q+} = \text{Tr}[\sigma_q R_n(E) + \sigma_{e\pm} T_n(E)]/(2N)^2$. Clearly the $n > 0$ (or $n < 0$) terms represent photon absorption (emission) terms. Although there is an infinite summation in Eq. (16), $G_n'(E)$ is at least of the order of pumping amplitude $(V_p)^n$ [see Eq. (13)] and hence $I_L^{q+}$ is of the order of $(V_p)^{2n}$. In the following numerical calculation, we keep terms $n$ up to 3 in summation. We have shown numerically that the higher-order terms are negligible. We also assume that the temperature is zero in the numerical calculation. For another limiting case, the adiabatic regime of parametric pumping concerns the $\omega \rightarrow 0$ limit. Under the adiabatic approximation, transforming Eq. (12) to the Wigner representation and using the Fisher-Lee relation, the expression recovers the known result in the adiabatic regime obtained from scattering matrix theory.\cite{28,30,38,39}

### III. ANALYTIC RESULT

Before presenting the numerical result, we first give some general results in the parametric pumping when pumping parameters are in phase in the presence of Rashba interaction. This means that if pumping parameters are of the form $V_p = V_p^0 \cos(\omega t + \phi)$, then \phi is independent of $i$. Without loss of generality, we assume that the pumped current is along the $x$ direction. First of all, from Eq. (7), we have ($\hbar = 1$ and $q = 1$)

$$ \sum_{\alpha} I_{\alpha q} = -\frac{1}{(2\pi)^2} \text{Tr} \left[ G^{q+} T G^\dagger T - G G^\dagger G^\dagger \right] \sigma_{q\sigma} \quad (17) $$

where $G'$ again is the retarded Green’s function with matrix element $G'(E,E')$, and $T_{\alpha\beta}$ denotes the trace over orbital degree of freedom. From Eq. (17), we can only conclude that the charge current is conserved, i.e.,

$$ \sum_{\alpha\sigma} I_{\alpha\sigma} = 0. \quad (18) $$

In general, the spin current may not be conserved. We now present some analytic results.

(i) If the potential including pumping potential (i.e., the full Hamiltonian excludes the kinetic energy and SO interaction) has the following symmetry:

$$ V(x,y,t) = V(-x,y,t) \quad (19) $$

then
\[ I_{L\alpha_x}(t) = I_{R\alpha_x}(t), \]
\[ I_{L\alpha_y}(t) = I_{R\alpha_y}(t), \]
\[ I_{L\alpha_z}(t) = I_{R\alpha_z}(t), \]
\[ \quad (20) \]

where \( I_{\alpha \pm} \) is the charge current with spin along the \( z \) direction, and if \( \alpha_z = \uparrow \), then \( \alpha_z = \downarrow \). To prove it, we make the following unitary transformation: \( z \rightarrow -z \), \( x \rightarrow -x \), \( \alpha_z \rightarrow -\alpha_z \), \( \alpha_y \rightarrow -\alpha_y \), \( \alpha_x \rightarrow -\alpha_x \), such that the Hamiltonian including the pumping potential is invariant under this transformation. Under this transformation, \( \alpha_{x,z} \) becomes \( -\alpha_{x,z} \) and the current \( I_L \) becomes \( I_R \) due to the change from \( x \) to \(-x \). This proves Eq. (20).

From the conservation of charge current Eq. (18), Eq. (20) says that if the system has a symmetry satisfying Eq. (19), then there is no pumped charge current. However, there may be spin current with spin along the \( x, y, \) and \( z \) directions. Indeed, as we will see in the following numerical calculation, the pumped spin current is nonzero. This suggests a robust way to generate pure spin current without accompanying charge current. No tuning parameters are needed. Now we examine the consequence of Eq. (20) on spin current. \( I_L - I_R \). From the first relation in Eq. (20), we have \( I_{L\alpha x} = I_{R\alpha x}, I_{L\alpha y} = I_{R\alpha y}, \) or \( I_{L\alpha z} = I_{R\alpha z} \). This means that during the pumping process, the pumped spin current with spin along the \( x \) direction is pumped into (or from) both leads. From a similar argument, the second and third relations of Eq. (20) give the conservation of spin current \( I_{L\alpha} + I_{R\alpha} = 0 \) for the spin along the \( y \) and \( z \) directions.

(ii) If the potential including the pumping potential has the following symmetry:

\[ V(x,y,t) = V(-x,-y,t), \]

Then

\[ I_{\alpha x}(t) = I_{\alpha x}(t), \]
\[ I_{\alpha y}(t) = I_{\alpha y}(t), \]
\[ I_{\alpha z}(t) = I_{\alpha z}(t), \]
\[ \quad (22) \]

Equation (22) says that if the system has a symmetry satisfying Eq. (21), then there is no pumped spin current with spin along the \( x \) and \( z \) directions. However, there may be charge current and spin current with spin along the \( y \) direction. To prove it, we make the following unitary transformation: \( z \rightarrow -z \), \( y \rightarrow -y \), \( \alpha_z \rightarrow -\alpha_z \), \( \alpha_y \rightarrow -\alpha_y \), \( \alpha_x \rightarrow -\alpha_x \), such that the Hamiltonian including the pumping potential is invariant under this transformation. Under this transformation, \( \alpha_{x,z} \) becomes \( -\alpha_{x,z} \) and we arrive then at Eq. (22).

(iii) If the potential including pumping potential has the following symmetry:

\[ V(x,y,t) = V(-x,-y,t), \]

then

\[ I_{L\alpha x}(t) = I_{R\alpha x}(t), \]
\[ I_{L\alpha y}(t) = I_{R\alpha y}(t), \]
\[ I_{L\alpha z}(t) = I_{R\alpha z}(t). \]
\[ \quad (24) \]

Combining Eq. (24) with the conservation of charge current, we see that if the system has a symmetry satisfying Eq. (23), then there is no pumped charge current. However, there may be pumped spin current during the pumping process. Equation (24) also says that the spin current with spin along the \( x \) and \( y \) directions is conserved and the spin currents with spin along the \( z \) direction from both leads are flowing into or out of the quantum pump. To prove Eq. (24), we make the following unitary transformation \( y \rightarrow -y \), \( x \rightarrow -x \), \( \alpha_y \rightarrow -\alpha_y \), and \( \alpha_x \rightarrow -\alpha_x \), such that the Hamiltonian including the pumping potential is invariant under this transformation. Under this transformation, \( \alpha_{x,z} \) becomes \( -\alpha_{x,z} \) and the current \( I_L \) becomes \( I_R \).

(iv) If the symmetry of a system satisfies both Eqs. (19) and (21), then there is no charge current and spin current with spin along the \( x \) and \( z \) directions. However, there may be spin current with spin along the \( y \) direction. As we know that in the limit \( \omega \) goes to zero, we arrive at the adiabatic limit. It has been shown that if the pumping parameters are in phase, the pumped current (charge or spin) calculated from the adiabatic formula must be zero.\(^{38}\) Therefore, the pumped charge and spin currents discussed above must come from the nonadiabatic contribution. We will verify this numerically in the next section.

IV. NUMERICAL RESULTS

Now we present the numerical results. To investigate the resonant behavior,\(^{46}\) we put a double-barrier confining potential \( U_0 \) in the central scattering region with Rashba SO coupling, where \( U_0 = V_0[\delta(x+d/2)+\delta(x-d/2)] \) with \( V_0 \) the barrier height and \( d \) the barrier separation. The time-dependent parametric pumping potential \( H'(t) = [V_y(x)\cos(\omega t)\delta(x+d/2)+V_y(x)\cos(\omega t+\delta)\delta(x-d/2)] \) is applied at the two edges. We first study the charge current and spin current along the \( x \) direction with spin along the \( z \) direction. According to analytic results, there may be pure spin current with spin along the \( z \) direction when the system satisfies Eqs. (19) and (23). We will check it in this section. The energy unit is set as 22.4 meV.\(^{41}\) In the numerical calculation, we choose \( U_0 = 1, V_0 = 0.5, d = 200 \) nm, and \( t_S = 0.1t \). We have discretized our system on a \( 40 \times 40 \) mesh. Note that the gate voltage is around 10 meV. Hence the changes of Rashba SO interaction induced by variation of gate voltage is small and can be neglected. Figure 1 shows pumped charge and spin currents as a function of the phase difference with \( V_y(y) = V_0\theta(y+0.5d-\theta(y)) \) for \( E_z = 0.0515 \) and \( \omega = 0.001 \). We see that at \( \phi = 0, 2\pi \), the pumped charge current is zero as expected from our analytic results since we have the symmetry \( V(x,y,t) = V(-x,y,t) \). On the other hand, we do have the pumped spin current at \( \phi = 0, 2\pi \). The pumped charge current as a function of phase difference behaves very much like
a sine function. For the pumped spin current, the minimum occurs near $\phi = \pi$. The phenomenon has been seen before and is attributed to the quantum interference between two photon-assisted processes occurring near pumping potentials $V_i$ and $V_j$.

Now we study the pumping potential that has the following symmetry:

$$V(x, y) = V(-x, -y).$$

In particular, in the following numerical calculation, we will set the pumping potential $V$ in two regions $x_i = -d/2, -d/2 \leq y_i \leq 0$ and $x_j = d/2, 0 \leq y_j \leq d/2$. Figure 2 shows the schematic plot of our setup. In Fig. 3, we plot the transmission versus Fermi energy. We see typical resonant tunneling behavior. In Fig. 4, we plot pumped charge current and spin current as a function of Fermi energy with phase difference $\phi = \pi/2$ and $\omega = 0.001$. Both charge and spin current are pumped out of the system. The pumped charge current and spin current show clear resonance behaviors that are well correlated with the transmission coefficient. Due to the existence of resonant states in the double-barrier structure, large pumped charge and spin currents are found near the resonance. This means that a spin-polarized current pumps out near-resonant energies. We also observe that the pumped charge current is positive while the pumped spin current can change sign as the Fermi energy is varied.

Now we investigate the spatial interference of the quantum pump by examining the following pumping potential at $x_i = -d/2, -d/2 \leq y_i \leq y$ and $x_j = d/2, -y \leq y_j \leq d/2$. Figure 5 presents the pumped charge current and spin current as a function of the position $y$. Here we have fixed $E_F = 0.0515$, $\omega = 0.001$, and $\phi = \pi/2$. We see that the pumped charge and spin current are very small near $y = -d/2$. When $y = 0$, the pumped charge current reaches the largest value in the positive direction and then decreases to zero when $y$ increases. When $y = d/2$, the charge current is maximum in the negative direction while the pumped spin current vanishes. This indicates that spatial interference plays an important role in the
generation of pumped spin current. Since the phases of pumping potential can be easily controlled experimentally, in Fig. 6 we show that the pumped charge and spin current as a function of phase difference for our system with pumping potential satisfies Eq. (14). We see that the behavior is very similar to Fig. 1. Again, the pure spin current can be generated at $\phi = 0$ due to the fact that the symmetry of the system satisfies Eq. (23).

Since pumped spin current can be generated at $\phi = 0$, it is natural to ask whether we can pump a spin current with only one pumping parameter. For this purpose, we set $V_j = 0$ and calculate the pumped charge current and spin current. In Fig. 7, we present pumped charge current and spin current as a function of Fermi energy with only one pumping potential at finite pumping frequency $\omega = 0.001$. We see that the magnitude of pumped charge current is larger than that with two driving forces while the pumped spin current is comparable to the two parameters pumping. Both the pumped charge current and spin current show resonant behaviors. At certain energies $E_F$, we have vanishing pumped charge current with nonvanishing spin current. Figure 8 presents pumped charge current and spin current as a function of the position $y$ where the single pumping potential is added. Similar to Fig. 5, we see that the pumped spin current vanishes when $y = d/2$. So the spatial interference is essential to pump out a spin-polarized current. In Fig. 9, we plot the pumped charge current and spin current versus pumping frequency. We see that the pumped charge current changes its direction a at certain frequency. This indicates that the pumped charge current can be controlled by the phase of the pumping potential.
current can be tuned to zero. Hence a pure spin current can be achieved by tuning the pumping frequency. Figure 10 shows pumped charge current and spin current as a function of pumping amplitude, which show monotonic dependence. We have verified that spin pump with two pumping potentials exhibits similar behaviors to those of Figs. 9 and 10.

Our numerical results show that in general, both pumped charge current and spin current are generated during the pumping process. A way to eliminate the pumped charge current is to add an external bias in the leads.39 In the presence of external bias in the leads, the charge current and spin current in the nonadiabatic regime can be rewritten as

\[ I_{0}^{\text{e/s}} = I_{0}^{\text{e/s}} + I_{p}^{\text{e/s}}, \]

where \( I_{0}^{\text{e/s}} \) is the charge current (spin current) due solely to external bias in the leads, corresponding to the \( n = 0 \) term in Eq. (12), and \( I_{p}^{\text{e/s}} \) is the charge current (spin current) due to the pumping effect, corresponding to nonzero \( n \) terms in Eq. (12). We set external bias in the left lead \( V_{L} = -b/\omega \) and external bias in the right lead \( V_{R} = b/\omega \). For illustration purposes, we have multiplied the charge current by a factor of 0.1.

In summary, we have investigated nonadiabatic parametric spin pumps at finite pumping amplitude and finite pumping frequency. Some general results concerning the symmetry of the parametric quantum pump in the presence of Rashba interaction are given. These results suggest a robust way of generating pure spin current without accompanying charge current. We find that spatial interference and photon-assisted tunneling play an important role in the pumping of spin current. For spin pump with two pumping potentials, the pure spin current can be achieved by tuning the phase difference, pumping frequency, as well as external bias.

**ACKNOWLEDGMENTS**

We gratefully acknowledge support by a RGC grant from the SAR Government of Hong Kong under Grant No. HKU 7044/05P, the CRCG grant from The University of Hong Kong, and support from NSFC under Grants No. 10574093 and No. 10274052 (Y.D.W.). Some of the calculations were performed at the Center for Computational Science, Hefei Institute of Physical Science.
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34. \( V_{xy} \) is a diagonal matrix with matrix element \( V_{xy} \exp(i\phi_{xy}) \), where \( xy \) labels the position \((x, y)\), \( V_{xy} \) is the pumping amplitude, and \( \phi_{xy} \) is the phase.


36. Note that this term is related to but not equal to the reflection coefficient.


41. Typically for GaAs, \( m = 0.068m_0 \), where \( m_0 \) is the electron bare mass and lattice parameter \( a_0 = 5 \text{ nm} \). The tight-binding constant \( t = \hbar^2 / 2m_0a_0^2 \) is chosen as energy unit, corresponding to \( \omega = 3.4 \times 10^{13} \text{ Hz} \).

